Quintessential solution of dark matter rotation curves and its simulation by extra dimensions

V.V.Kiselev

State Research Center "Institute for High Energy Physics"
Protvino, Moscow region, 142281 Russia
Fax: +7-0967-744739, E-mail: kiselev@th1.ihep.su

Abstract

On the base of an exact solution for the static spherically symmetric Einstein equations with the quintessential dark matter, we explain the asymptotic behavior of rotation curves in spiral galaxies. The parameter of the quintessence state, i.e. the ratio of its pressure to the density is tending to $-1/3$. We present an opportunity to imitate the relevant quintessence by appropriate scalar fields in the space-time with extra 2 dimensions.

1 Introduction

The rotation curves in spiral galaxies, i.e. the dependence of rotation velocity on the distance from the center of galaxy, as observed astronomically are typically given by the profile represented in Fig. 1. This picture shows that the contribution determined by the visible matter (the dotted line) is falling down beyond the optical size of the galaxy ($R_{\rm opt}$) in agreement with the behavior expected from the Newton's law for the gravity force of point-like mass, while in the central disk of galaxy this term is decreased with the decrease of mass involved in the interaction. The observed curves prove the presence of dark halo causing the flat, non-falling character of rotation curves in the asymptotic region beyond the optical size. The corresponding term shown by the dashed line begins to dominate at large distances.

The described superposition of two contributions allows a phenomenological description in terms of universal rotation curves [2]. The dark matter review can be found in [3].

As for the explanations of the halo dominated contribution [4], we emphasize the attempt of [5] to build the dynamics in terms of scalar fields of the quintessence kind [6], so that the flat rotation curves were found to be the results of quintessence with the pressure-to-density ratio close to $-1/3$.

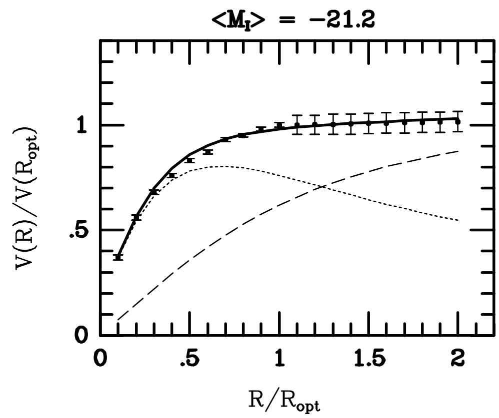

Figure 1: The characteristic rotation curve taken from [1].

In this paper we apply our recent result on the exact solution of spherically symmetric static Einstein equations with the perfect fluid of quintessence [7] to the problem of rotation curves in the asymptotic region of dark-matter-halo dominance. This class of solutions agrees with the common consideration of static metrics given in [8]. We find an exact description of asymptotic behavior in terms of the quintessence and give its interpretation by the scalar fields in extra 2 dimensions.

2 Exact results

In this section, we, first, derive the relation between the metric components and the rotation velocity. Second, we show how the quintessential solution reproduces the asymptotic behavior of rotation curves in the halo dominant region. Third, we explore the adiabatic approximation in order to describe some variation in the pre-asymptotic region. Fourth, we give the interpretation of obtained results in terms of scalar fields in the extra dimensions.

2.1 Rotation equations

We describe the rotation curves in the halo dominated region in the framework of Hamilton–Jacobi formalism. So, let us consider the equation for the motion of a test particle with a mass m in the gravitational field,

$$g^{\mu\nu}\,\partial_\mu S\,\partial_\nu S - m^2 = 0 \tag{1}$$

with the metric yielding the interval

$$\mathrm{d}s^2 = g_{tt}(r)\,\mathrm{d}t^2 - \frac{1}{g_{tt}(r)}\,\mathrm{d}r^2 - r^2[\mathrm{d}\theta^2 + \sin\theta^2\mathrm{d}\phi^2],$$

which belongs to the class relevant to the problem under interest. Following the general framework, we write down the solution in the form, which incorporates two integrals of the motion in the spherically symmetric static gravitational field,

$$S = -\mathcal{E}\,t + \mathfrak{M}\,\theta + \mathcal{S}(r), \tag{2}$$

where $\mathcal{E}$ and $\mathfrak{M}$ are the conserved energy and rotational momentum, respectively. Then, from (1) we deduce

$$\left(\frac{\partial \mathcal{S}}{\partial r}\right)^2 = \frac{1}{g_{tt}^2}\mathcal{E}^2 - \frac{1}{g_{tt}}\left(\frac{\mathfrak{M}^2}{r^2} + m^2\right), \tag{3}$$

which results in

$$\mathcal{S} = \int\limits_{r_0}^{r(t)} \mathrm{d}r\, \frac{1}{g_{tt}(r)}\sqrt{\mathcal{E}^2 - V^2(r)}, \tag{4}$$

where V^2 is an analogue of potential,

$$V^2(r) = g_{tt}(r)\left(\frac{\mathfrak{M}^2}{r^2} + m^2\right).$$

The trajectory is implicitly determined by the equations

$$\frac{\partial S}{\partial \mathcal{E}} = \mathsf{const} = -t + \int\limits_{r_0}^{r(t)} \mathrm{d}r\, \frac{1}{g_{tt}(r)}\frac{\mathcal{E}}{\sqrt{\mathcal{E}^2 - V^2(r)}}, \tag{5}$$

$$\frac{\partial S}{\partial \mathfrak{M}} = \mathsf{const} = \theta - \int\limits_{r_0}^{r(t)} \mathrm{d}r\, \frac{1}{r^2}\frac{\mathfrak{M}}{\sqrt{\mathcal{E}^2 - V^2(r)}}. \tag{6}$$

Taking the derivative of (5) and (6) with respect to the time, we get

$$1 = \dot{r}\,\frac{\mathcal{E}}{g_{tt}\sqrt{\mathcal{E}^2 - V^2(r)}}, \tag{7}$$

$$\dot{\theta} = \dot{r}\,\frac{\mathfrak{M}}{r^2\sqrt{\mathcal{E}^2 - V^2(r)}}, \tag{8}$$

and, hence,

$$\mathcal{E} = \frac{g_{tt}}{rv}\,\mathfrak{M}, \tag{9}$$

relating the energy and the rotational momentum, where we have introduced the velocity

$$v \stackrel{\text{def}}{=} r\dot{\theta}.$$

The points of return are determined by

$$\dot{r} = 0, \quad \Rightarrow \quad \mathcal{E}^2 - V^2 = 0, \quad \Rightarrow \quad \mathfrak{M}^2 = m^2 r^2 \frac{v^2}{g_{tt} - v^2}.$$

The circular rotation takes place, if two return points coincide with each other, i.e. we have the stability of zero $\dot{r}$ condition. Introducing the proper distance λ by

$$\frac{\partial}{\partial \lambda} = \frac{\partial r}{\partial \lambda}\,\frac{\partial}{\partial r} = g_{tt}\,\frac{\partial}{\partial r}$$

we deduce the equation

$$\left(\frac{\partial \mathcal{S}}{\partial \lambda}\right)^2 = \mathcal{E}^2 - V^2, \tag{10}$$

so that the stability of circular motion implies the stability of potential,

$$\frac{\partial V^2}{\partial r} = 0. \tag{11}$$

Then, we get

$$v^2 = \frac{1}{2}\,\frac{\mathrm{d}g_{tt}(r)}{\mathrm{d}\ln r}. \tag{12}$$

This result is in agreement with the nonrelativistic approximation. Indeed, in the Newton's limit, the equality of two forces, the gravitational attraction and the circular rotation inertia, gives

$$F_{NR} = \frac{m}{2}\,\frac{dg_{tt}(r)}{dr} = \frac{mv^2}{r},$$

which reproduces the exact result of (12).

Introducing a re-scaled velocity with respect to the proper time,

$$\mathfrak{v}^2 = \frac{1}{g_{tt}}\,v^2,$$

we get the result of [9]

$$\mathfrak{v}^2 = \frac{1}{2}\,\frac{\mathrm{d}\ln g_{tt}(r)}{\mathrm{d}\ln r}.$$

Thus, we have determined the exact profile of circular rotation curves for the metric under interest.

2.2 Quintessential solution

For the spherically symmetric static metric

$$\mathrm{d}s^2 = g_{tt}(r)\,\mathrm{d}t^2 + g_{rr}(r)\,\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2),$$

we have found the exact solution [7] with

$$g_{tt} = 1 - \sum_n \left(\frac{r_n}{r}\right)^{3w_n+1}, \tag{13}$$

where r_n are positive constants, and

$$g_{rr} = -\frac{1}{g_{tt}},$$

so that the matter averaged over the angles satisfies the perfect fluid relation between the pressure and energy density with the state parameter w_n,

$$p_n = w_n \rho_n.$$

The components of energy-momentum tensor are given by

$$T^{[n]}{}_t{}^t = \rho_n(r), \tag{14}$$

$$T^{[n]}{}_i{}^j = \rho_n(r)\,3w_n\left[-(1+3\,B_n)\frac{r_i\,r^j}{r_k r^k} + B_n\,\delta_i{}^j\right], \tag{15}$$

so that the averaging gives

$$\left\langle T^{[n]}{}_{i}{}^{j} \right\rangle = -p_n(r)\,\delta_i{}^j,$$

independently of the parameter B. However, the Einstein equations are satisfied at the appropriate value of

$$B_n = -\frac{3w_n+1}{6w_n}, \tag{16}$$

which is the only parametrization consistent with the superposition of various terms. In addition, the above class of exact solutions covers the characteristic limits such as the case of a collapsed dust, i.e. a black hole, at $w_0 = 0$, the vacuum solution, i.e. de Sitter space, at $w_{-1} = -1$, and the relativistic electromagnetic field of charged black hole at $w_{em} = 1/3$.

The superposition implies that the sum of terms in the time component of the metric is exactly transformed into the sum of energy-momentum tensors,

$$\sum_n \left(\frac{r_n}{r}\right)^{3w_n+1} \quad\Longrightarrow\quad \sum_n T^{[n]}{}_{\mu}{}^{\nu},$$

so that we can get various exact spherically symmetric static solutions of Einstein equations by combinations of relevant terms.

Let us consider the limit of quintessence with the state parameter $w_q = -1/3+\epsilon \to -1/3+0$. Then, the metric component

$$\tilde{g}_{tt} = 1 - \frac{\alpha}{\epsilon}\left[\left(\frac{r_q}{r}\right)^{\epsilon} - 1\right]$$

tends to

$$\tilde{g}_{tt} = 1 + \alpha \ln\frac{r}{r_q}, \tag{17}$$

and in accordance with eq.(12) we find the rotation velocity

$$v^2 = \frac{1}{2}\,\alpha, \tag{18}$$

describing the asymptotic behavior at large distances, i.e. in the halo dominated region. Thus, the quintessential solution describes the asymptotic rotation curves with the metric (17) for the dark matter.

Numerically, the quantity α is of the order of 10^{-6}, which implies that the inner horizon is posed at a distance many orders of magnitude less than the parameter r_q.

The energy-momentum tensor for the quintessence is given by the expression

$$T^{[q]}{}_{t}{}^{t} = T^{[q]}{}_{r}{}^{r} = \rho_q(r) = -\frac{\alpha}{2r^2}\left(\ln\frac{r}{r_q}+1\right), \tag{19}$$

$$T^{[q]}{}_{\theta}{}^{\theta} = T^{[q]}{}_{\phi}{}^{\phi} = -\frac{\alpha}{4r^2}. \tag{20}$$

Averaging over the angles results in the parameter of state equation equal to

$$w_q = -\frac{1}{3}\left(1+\frac{1}{\ln\frac{r}{r_q}+1}\right), \tag{21}$$

which has a singular point at $\ln r/r_q = -1$ (see Fig. 2).

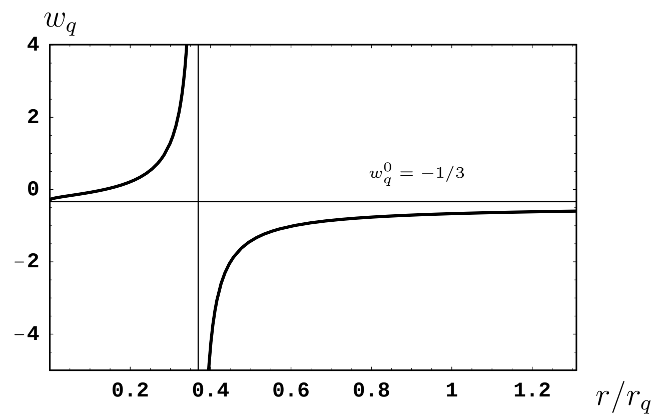

Figure 2: The parameter of state equation for the quintessence.

To the same moment we can isolate two parts in the energy density, i.e. the logarithmic contribution and the $1/r^2$-term, so that their state parameters are equal to

$$w_{\ln} = -\frac{1}{3}, \quad w_{1/r^2} = -\frac{2}{3},$$

respectively. However, this separation is not unique, and any arbitrary redefinition of r_q parameter will result in the rearrangement. For example, introducing a large scale $\ln \tilde{r}_q/r_q$, we get

$$\tilde{w}_{\ln} = -\frac{1}{3}, \quad \tilde{w}_{1/r^2} = -\frac{1}{3}\,\frac{2+\ln \tilde{r}_q/r_q}{1+\ln \tilde{r}_q/r_q} \to -\frac{1}{3}, \quad \text{at} \quad \ln \tilde{r}_q/r_q \to \infty,$$

which is the case we have been going to consider with no singularity of the constant w.

The spatial part of energy-momentum tensor for the logarithmic term is purely radial, while the $1/r^2$-term is tending to the radial form with the small contribution isotropic over the angles.

Thus, we can draw a conclusion on the dark matter described by the quintessence with a negative pressure at $w_q = -1/3$ corresponds to the asymptotically flat rotation curves, so that the exact solution of static spherically symmetric Einstein equations allows the superposition of various terms such as the Schwarzschild black hole surrounded by the quintessence.

2.3 Adiabatic modification

Let us consider a phenomenologically motivated variation of the parameter determining the pre-asymptotic behavior of the term contributing to the star velocities due to the dark matter halo,

$$\alpha = \alpha_0\,\frac{r^2}{a^2+r^2},$$

where the characteristic scale is close to the optical size of the galaxy, $a \sim R_{\rm opt}$.

An adiabatic variation of parameter α implies that the variation of energy density caused by a small change of the parameter versus the distance is much less than the variation due to the leading dependence on the distance. Therefore, we demand the condition of adiabatic approximation in the form

$$\left|\frac{\partial\rho}{\partial\alpha}\,\frac{\partial\alpha}{\partial r}\right| \ll \left|\frac{\partial\rho}{\partial r}\right|.$$

In the metric under study we get

$$\left|\frac{\partial \ln \alpha}{\partial \ln r}\right| \ll \left|\frac{\ln \frac{r}{r_q}+1}{\ln \frac{r}{r_q}}\right|.$$

The phenomenological parametrization of α gives

$$\frac{2a^2}{a^2+r^2} \ll \left|\frac{\ln \frac{r}{r_q}+1}{\ln \frac{r}{r_q}}\right|,$$

so that the adiabatic approximation is sound at $r \gg a$, while at $r \sim a$ we deduce the condition

$$r_q \sim a,$$

which implies that the quintessence-term parameter r_q is determined by the optical size of the galaxy, if we desire to incorporate the observed decrease of the dark matter contribution into the rotation curves within the description suggested above. We present the dependence of angle

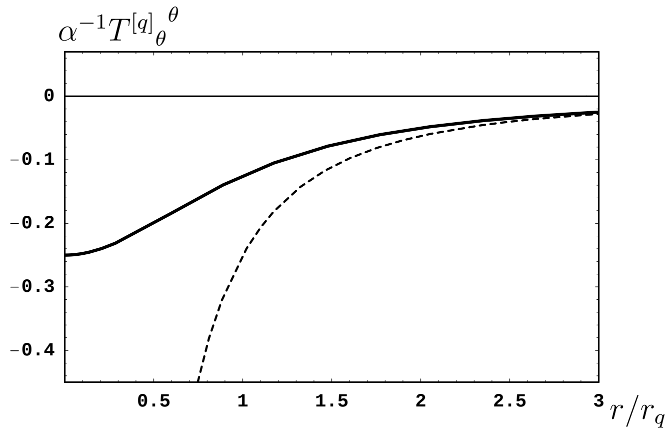

Figure 3: The adiabatic regularization of the angle component in the energy-momentum tensor of quintessence at $a = r_q$ (the solid curve) in comparison with the constant α (the dashed line).

component for the energy-momentum tensor of the quintessence under the adiabatic change of the parameter α in Fig. 3.

Thus, the decrease of rotation velocity at the distances less than the optical size of galaxy as caused by the dark matter can be included in the offered mechanism by the small adiabatic change of the solution parameter.

2.4 Interpretation

The quintessential state with the negative pressure is usually considered as a perfect fluid approximation of a scalar field with an appropriate potential. So, let us start with a general contribution of the scalar field.

A scalar field φ with the lagrangian equal to

$$\mathcal{L} = \frac{1}{2}\, g^{\mu\nu}\, \partial_\mu\varphi\, \partial_\nu\varphi - V(\varphi), \tag{22}$$

generates the energy-momentum tensor

$$T_{\mu\nu} = \partial_\mu \varphi \, \partial_\nu \varphi - g_{\mu\nu} \left[\frac{1}{2} \, g^{\beta\gamma} \, \partial_\beta \varphi \, \partial_\gamma \varphi - V(\varphi) \right]. \tag{23}$$

The static field $\varphi(r)$ depending on the distance, gives

$$T_t^{\ t} = T_\theta^{\ \theta} = T_\phi^{\ \phi} = \rho(r) = -\mathcal{L}, \tag{24}$$

$$T_r^{\ r} = T_t^{\ t} - g^{tt} [\partial_r \varphi(r)]^2. \tag{25}$$

Eqs. (24) and (25) compared with eqs. (19) and (20) imply that the explanation of the quintessential solution for the rotation curves in the region of dark matter dominance in terms of the scalar field requires some additional contributions to the time component of the energy-momentum tensor, i.e. we should, at least, suppose the presence of a cold dark matter with the density

$$\rho_{CDM} = -g^{tt} [\partial_r \varphi(r)]^2,$$

which is not natural, since the density is negative, and it is amazingly coherent with the scalar field.

Thus, we do not impose the usual scalar version of quintessence suitable for the purposes of describing the rotation curves as caused by the dark matter.

Let us consider the space-time with two extra dimensions, so that the metric is determined by the interval

$$\mathrm{d}s^2 = g_{tt}(r) \, \mathrm{d}t^2 - \frac{1}{g_{tt}(r)} \, \mathrm{d}r^2 - r^2 [\mathrm{d}\theta^2 + \sin\theta^2 \mathrm{d}\phi^2] + \kappa(y_{-1}) \, \mathrm{d}y_{-1}^2 - \kappa(y_4) \, \mathrm{d}y_4^2, \tag{26}$$

and the 4 dimensional (4D) interval is given by the condition

$$\mathrm{d}y_{-1}^2 = \mathrm{d}y_4^2.$$

Introduce two scalar fields by the following definitions:

$$\bar{\varphi} = e^{y_{-1}}, \tag{27}$$

which is the isotropic function, and the triplet

$$\varphi^{(1)} = e^{y_4} \sin\theta \, \cos\phi, \tag{28}$$

$$\varphi^{(2)} = e^{y_4} \sin\theta \, \sin\phi, \tag{29}$$

$$\varphi^{(3)} = e^{y_4} \cos\theta, \tag{30}$$

depending on the angles.

Then, under the condition of

$$\kappa(y_{extra}(r)) = r^2,$$

for the metric components restricted to the 4D world, we find for the energy-momentum tensors the following expressions:

$$\bar{T}_t^t = \bar{T}_r^r = \bar{T}_\theta^\theta = \bar{T}_\phi^\phi = -\mathcal{L}(\bar{\varphi}) = -\frac{1}{2r^2} \, e^{2y_{-1}} + \bar{V}(\bar{\varphi}), \tag{31}$$

$$T_t^{\ t} = T_r^{\ r} = -\mathcal{L}(\varphi^{(i)}) = \frac{3}{2r^2} \, e^{2y_4} + V(\varphi^{(i)}), \tag{32}$$

$$T_\theta^{\ \theta} = T_\phi^{\ \phi} = \frac{1}{2r^2} \, e^{2y_4} + V(\varphi^{(i)}). \tag{33}$$

The 4D space-time is considered at

$$y_{-1} = y_4, \tag{34}$$

so that at

$$\bar{V} = V$$

we can easily get that the scalar fields simulate the energy-momentum tensor of quintessence, if we put

$$\begin{aligned} \bar{\varphi}^2 &= \left[\varphi^{(i)}\right]^2 = -\frac{\alpha}{4}\left(2\ln\frac{r}{r_q}+1\right), &(35)\\ 2V &= -\frac{\alpha}{4r_q^2}\exp\left[1+\frac{4}{\alpha}\bar{\varphi}^2\right] = -\frac{\alpha}{4r^2}. &(36)\end{aligned}$$

In order to avoid a conflict caused by the sign of distance-dependent term in eq.(35), we make the re-scaling of the initial lagrangian for the scalar fields by

$$\hat{\mathcal{L}} = \mathcal{L}\cdot\mathcal{K}(\varphi), \tag{37}$$

where, for definiteness, we put

$$\mathcal{K} = \frac{1}{\sqrt{\bar{\varphi}^2}}.$$

Then, with the same condition on the extra coordinates we derive

$$\begin{aligned} \hat{\bar{\varphi}}^2 &= \left[\hat{\varphi}^{(i)}\right]^2 = e^{2y_{-1}(r)} = \frac{\alpha^2}{16}\left(2\ln\frac{r}{r_q}+1\right)^2, &(38)\\ 2V\cdot\mathcal{K} &= -\frac{\alpha}{4r_q^2}\exp\left[1-\frac{4}{\alpha}\sqrt{\hat{\bar{\varphi}}^2}\right] = -\frac{\alpha}{4r^2}, &(39)\\ \mathcal{K}^{-1} &= -\frac{\alpha}{4}\left(2\ln\frac{r}{r_q}+1\right). &(40)\end{aligned}$$

We emphasize that the extra dimensional positions of our 4D world, i.e. the functions $y_{extra}(r)$, are twofold. The same note should be done on the factor $\mathcal{K}$ (twofold values of the square root). The sign of kinetic term for the scalar fields changes with the sign of $\mathcal{K}$. Therefore, we deal with two branches of the field variation, representing the normal and ghost phases[1]. Nevertheless, the energy density remains finite.

Thus, we have just shown that the energy-momentum 4D tensor of quintessence responsible for the flat asymptotic form of rotation curves in spiral galaxies, i.e. the dark matter contribution, can be exactly imitated by the scalar fields with 2 extra dimensions.

In this paper we do not investigate the field equations in the extra dimensions. Note, first, that the corresponding components of the tensor $R_{-1\,-1} = R_{4\,4} = 0$, and they do not contribute to the 4D Einstein equations through the scalar curvature. Second, in the extra dimensions we can easily find that $R_{ab} - 1/2 g_{ab} R \neq -2T_{ab}$, where $\{a,b\} \in \{-1,4\}$, $R_{ab} = 0$, and T_{ab} is the energy-momentum tensor of scalar fields under consideration. Of course, we can add two scalar fields $\bar{\chi}(y_{-1})$ and $\chi(y_4)$, so that they have appropriate potentials producing $\mathcal{L}(\bar{\chi},\chi) = 0$, if the field

[1]The ghosts have negative sign in front of the kinetic term in the lagrangian. On a relevance of tachyons in the modern theory see review in [10].

equations are satisfied, and hence, χ's do not contribute to the 4D Einstein equations, while their derivatives are adjusted in order to make the Einstein equations valid in the extra dimensions. In that case the extra-dimensional components of energy-momentum tensor are proportional to the metric, i.e. the situation in extra dimensions looks like the vacuum solution with the curvature depending on the 3D distance as a parameter. So, the scalar field equations are topics of separate investigations. Thus, we claim only that the extra-dimensional scalar fields with appropriate exponential potentials simulate the quintessential solution for the rotation curves.

By the way, once we have encountered the problem of ghosts in the treatment with extra dimensions, we have to note that equations equivalent to (38)–(40) can be replicated in the 4D space-time, so that the only difference is the overall negative sign of the lagrangian for the triplet field[2].

The change of normal phase to the ghost for the scalar field is spectacular, since it is closely related to the variation of sign for the energy density. Both these facts are irrelevant in the case of adiabatic growth of the parameter α at $\ln r/r_q \approx 0$. The ghost phase is essential in the asymptotic region of large distances, if we do believe in the constant velocity of rotation in infinity. On another side, the negative value of kinetic energy is familiar from the quantum mechanics if the particle enters the classically forbidden region under the potential barrier.

3 Conclusion

In this paper we have found a quintessential solution for the problem on the asymptotic behavior of the rotation curves in spiral galaxies in the region of dark-matter-halo dominance. The explanation is constructed on the base of new class of metrics, describing the perfect fluid with a negative pressure in the static spherically symmetric gravitational field. This class satisfies the principle of superposition for various kind of matter contributions, and, hence, it does not destroys the Schwarzschild metric by adding some amount of exotic or ordinary matter.

The quintessence with the state parameter $w_q = -1/3$ exactly results in the flat limit of rotation curves. Its energy-momentum tensor is simulated by scalar fields in extra dimensions with appropriate exponential potentials.

The author thanks Prof. R.Dzhelyadin for the possibility to collaborate with the LHCB group at CERN, where this work has been done. I am grateful to Prof. S.S.Gershtein for useful discussions and valuable remarks, and to members of Russian team in LHCB for a kind hospitality.

This work is supported in part by the Russian Foundation for Basic Research, grants 01-02-99315, 01-02-16585, and 00-15-96645.

References

[1] K. A. Olive, arXiv:astro-ph/0301505.

[2] M. Persic, P. Salucci and F. Stel, Mon. Not. Roy. Astron. Soc. **281**, 27 (1996) [arXiv:astro-ph/9506004].

[3] S. Khalil and C. Munoz, Contemp. Phys. **43**, 51 (2002) [arXiv:hep-ph/0110122]; P. J. Peebles and B. Ratra, arXiv:astro-ph/0207347.

[2] We have also suggested that the squares of scalar fields depend on the radius but the extra coordinates.

[4] R. H. Sanders and S. S. McGaugh, arXiv:astro-ph/0204521;
A. Arbey, J. Lesgourgues and P. Salati, Phys. Rev. D **65**, 083514 (2002) [arXiv:astro-ph/0112324], Phys. Rev. D **64**, 123528 (2001) [arXiv:astro-ph/0105564], arXiv:astro-ph/0301533;
U. Nucamendi, M. Salgado and D. Sudarsky, Phys. Rev. Lett. **84**, 3037 (2000) [arXiv:gr-qc/0002001], Phys. Rev. D **63**, 125016 (2001) [arXiv:gr-qc/0011049].

[5] T. Matos, F. S. Guzman and D. Nunez, Phys. Rev. D **62**, 061301 (2000) [arXiv:astro-ph/0003398];
T. Matos, F. S. Guzman, L. A. Urena-Lopez and D. Nunez, arXiv:astro-ph/0102419;
T. Matos, F. S. Guzman and L. A. Urena-Lopez, Class. Quant. Grav. **17**, 1707 (2000) [arXiv:astro-ph/9908152];
F. S. Guzman, T. Matos, D. Nunez and E. Ramirez, arXiv:astro-ph/0003105.

[6] T. Chiba, Phys. Rev. D **60**, 083508 (1999) [arXiv:gr-qc/9903094];
N. A. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, Science **284**, 1481 (1999) [arXiv:astro-ph/9906463];
P. J. Steinhardt, L. M. Wang and I. Zlatev, Phys. Rev. D **59**, 123504 (1999) [arXiv:astro-ph/9812313];
L. M. Wang, R. R. Caldwell, J. P. Ostriker and P. J. Steinhardt, Astrophys. J. **530**, 17 (2000) [arXiv:astro-ph/9901388];
V. Sahni, Class. Quant. Grav. **19**, 3435 (2002) [arXiv:astro-ph/0202076].

[7] V. V. Kiselev, Class. Quant. Grav. **20**, 1187 (2003) [arXiv:gr-qc/0210040].

[8] S. Mignemi and D. L. Wiltshire, Class. Quant. Grav. **6**, 987 (1989);
S. Mignemi and D. L. Wiltshire, Phys. Rev. D **46**, 1475 (1992);
S. J. Poletti and D. L. Wiltshire, Phys. Rev. D **50**, 7260 (1994) [Erratum-ibid. D **52**, 3753 (1995)] [arXiv:gr-qc/9407021];
K. G. Zloshchastiev, Phys. Rev. D **64**, 084026 (2001) [arXiv:hep-th/0101075].

[9] S. Chandrasekhar, *Mathematical Theory of Black Holes* (Oxford Science Publications, Oxford, 1983).

[10] G. W. Gibbons, arXiv:hep-th/0301117;
A. Sen, JHEP **0207**, 065 (2002) [arXiv:hep-th/0203265].